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Fatigue crack growth in polyurethane foam

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The growth of fatigue cracks in compact tension specimens of rigid polyurethane foam has been studied at room temperature under conditions of constant load–amplitude cycling. The growth of the cracks at the frequencies employed (~ 0.2 Hz) is found to be reasonably reproducible and the growth rate can be related to the cyclic stress intensity range in the conventional way. The rate of growth of the cracks is also found to depend on the mean stress level and an attempt has been made to separate out the effects of stress-intensity range, ΔK, and the maximum stress intensity in each cycle, K_{max}, by combining the data obtained under a variety of loading conditions.

1. Introduction

Fatigue crack growth studies in polymeric materials have been carried out on a variety of different systems [1–4] and, in general, the analysis of the results has centred on the Paris growth equation [5] or on modified forms of this equation [6] which enable the effects of mean stress level to be taken into account. Investigations of this type have shown that this approach, largely developed in the context of metal fatigue, can be successfully applied to crack propagation in polymers though, as in the case of metals, the application is still largely empirical rather than mechanistic. Nevertheless, the crack growth rate parameters which are involved in the Paris equation or its modifications do provide a basis for the interpretation of fatigue crack behaviour and, of more immediate significance, provide a basis for designing against fatigue in structures which can be assumed to contain crack-like defects of determinable size.

With the increasing use of foamed plastics, and polyurethane foam in particular as load-bearing components in a wide variety of applications, the desirability of demonstrating the applicability (or otherwise) of the above approach to the fatigue behaviour of such materials is apparent. The selection of a polyurethane foam for such a study was motivated by its intended use as a load-bearing insulant in the transportation of liquid gases, a useage in which cyclic loading is inevitable and failure by fatigue a real possibility. Although such useage would normally involve service at sub-ambient temperatures, the outer surface of the insulant would be at ambient temperature and it is with crack growth at room temperature that this study is concerned.

2. Experimental procedure

The foam used in this work was supplied by Shell Research Ltd, in the form of large panels of the material, made by spraying. The panels were built up of successive layers about 10 mm thick, each layer corresponding to one pass of the spray gun and each being separated from the next by a thin "skin" or interlayer of high-density material about 0.15 mm thick. The density of the material between the interlayers was 85 kg m^{-3} and the foam in these regions (the bulk of the material) was of the closed cell type, an example of the structure of which is shown in Fig. 1. This photograph was taken by optical microscopy of a thin section of the material viewed in transmitted light. The cell diameter was estimated to be about 180 μm [7].

The crack-growth specimens which were cut from the supplied panels were of the compact tension type according to ASTM specification E399-74 and are shown, together with the relevant dimensions, in Fig. 2. Fig. 2 also shows the position of the crack starter notch relative to the foam interlayers. As far as possible this was positioned such that the fatigue crack would spread entirely through the low-density foam of the layers, mid-way between the high-density skins bounding the

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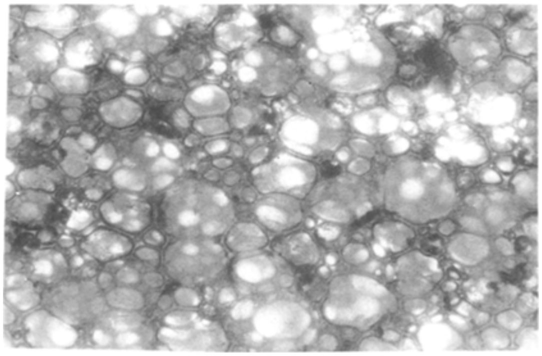

Figure 1 Optical transmission micrograph of the foam structure (× 40).

layers, to avoid the possibility of interference between the crack and the skins.

The specimens were tested by load cycling between pre-determined limits of tensile load using a screw-driven 'Instron' testing machine. Because of the mechanism of the machine it was not possible to adjust the cross-head velocity sufficiently finely to maintain a constant frequency from specimen to specimen when different load ranges were employed or as the specimen compliance varied as crack growth occurred. The frequencies however, were maintained as near constant as possible and fell within the range 0.15 to 0.25 Hz.

As the fatigue crack grew its length was measured on the front and back surfaces of the rectangular specimens by means of a millimeter scale and an (× 10) eye-glass and the readings were averaged. It was estimated that this technique enabled the position of the crack tip to be measured to within ± 0.2 mm and it was considered that greater optical refinement was unnecessary in view of the difficulty of defining the exact position of the tip in the cellular structure at the surface. The straightness of the crack-front was checked by the use of a dye to mark the position of the fatigue crack and then the specimen was broken in one stress application to check the geometry and position of the crack-front relative to the tip position indicated by the surface measurements. The results showed that the crack-front was reasonably straight and gave confidence that the surface measurements provided a representative indicator of the progress of the crack.

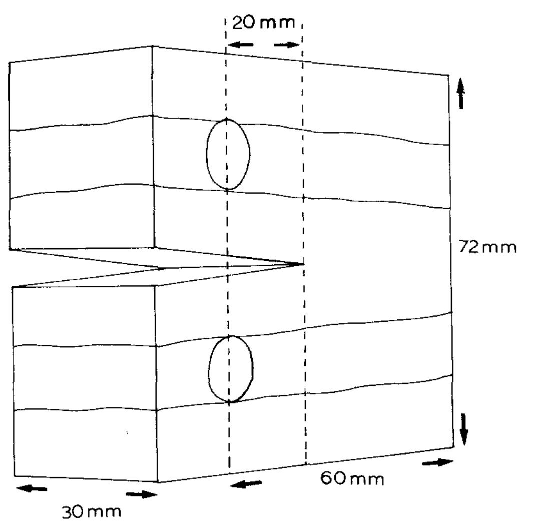

Figure 2 The compact tension specimen showing the position of the crack relative to the inter-layers of the foam.

3. Analysis of the crack-growth data

3.1. Analysis of individual crack-growth curves

The Paris equation for the rate of growth of a fatigue crack is written, in its simplest form:

$$\frac{da}{dN} = c\Delta K^m, \quad (1)$$

where a is the crack length, N is the number of load cycles, ΔK is the range of stress intensity experienced by the crack tip in each load cycle, and c and m are constants. If the maximum and minimum loads in each cycle are P_{max} and P_{min}, respectively, then ΔK can be expressed in terms of the load range ΔP $(P_{max} - P_{min})$ according to the relation:

$$\Delta K = \frac{\Delta P}{BW^{1/2}} \cdot \phi\left(\frac{a}{w}\right), \quad (2)$$

where B and w are the specimen dimensions normal to and parallel to the direction of the crack growth respectively and $\phi(a/w)$ is a polynomial of the form [8]

$$\phi\left(\frac{a}{w}\right) = 29.6\left(\frac{a}{w}\right)^{1/2} - 185.5\left(\frac{a}{w}\right)^{3/2} + 655.7\left(\frac{a}{w}\right)^{5/2} - 1017\left(\frac{a}{w}\right)^{7/2} + 639.9\left(\frac{a}{w}\right)^{9/2}.$$

The maximum stress intensity in any cycle is given by:

$$K_{max} = \frac{P_{max}}{BW^{1/2}} \cdot \phi\left(\frac{a}{w}\right) \quad (3)$$

and ΔK and K_{max} are related through the R-ratio (P_{min}/P_{max}) since $\Delta K/K_{max} = 1 - R$. In order to analyse the data in terms of the Paris equation the a against N data for each specimen were fitted to a smooth curve using a quadratic spline technique computer program developed by Shell Research Ltd, and values of the crack growth rate, da/dN, at selected values of a were obtained as a function of the appropriate ΔK for the crack length in question. These data were then plotted in the conventional way to produce a graph of log (da/dN) against log ΔK for each specimen and values of c and m were derived from the slope and intercept of the regression analysis line through the data points.

3.2. Analysis of growth-rate data for different conditions of loading

The disadvantage of constant load–amplitude tests performed on conventional compact tension specimens is that as the crack grows the resulting increase in ΔK is accompanied by an increase in K_{max} as well and so the effects of both these changing quantities on the crack growth rate are superimposed. In materials in which the K_{max} dependence is negligible this is not a serious limitation but, as will be seen in the next section, this is not the case here. It is, of course, possible in principle to separate out the K_{max} effect by comparing the values of c and m obtained from tests with different R-ratios but if, as found here, the scatter in these values is too large to permit a systematic variation to be discerned, an alternative method of approach may be more fruitful and this has been attempted in the present study.

The method adopted is based on a previously suggested [9, 10] modification of the Paris equation explicitly incorporating the effect of K_{max}, which has the form:

$$\frac{da}{dN} = c'\Delta K^m \cdot K_{max}^n, \quad (4)$$

c', m and n being constants. Since the tests involved in the present work are at constant load–amplitude it is appropriate to re-write this equation in terms of load rather than stress intensity, when (using Equations 2 and 3)

$$\frac{da}{dN} = c'\left[\frac{\phi(a/w)}{BW^{1/2}}\right]^{(m+n)} \Delta P^m \cdot P_{max}^n. \quad (5)$$

For a series of tests performed at a fixed value of ΔP but in which P_{max} is changed from test to test, the number of cycles required for crack growth to a given length, N_a, will vary from test to test and can be found by inverting and integrating Equation 5 which, since ΔP is constant, yields:

$$N_a = I_1 P_{max}^{-n}, \quad (6)$$

where I_1 includes the integral of $\phi(a/w)$ over the defined crack-growth increment (the same for each test), ΔP^m and c'. Thus a plot of $\log N_a$ against $\log P_{max}$ should be linear and should yield the value of n from its gradient.

Similarly, for a series of tests for which P_{max} is invariant but ΔP is changed from test to test, the number of cycles for crack growth to a prescribed length in each test, N_a, will be given by:

$$N_a = I_2 \Delta P^{-m}, \quad (7)$$

where I_2 differs from I_1 only in containing P_{max}^n as the constant load term, replacing ΔP^m. In this case, therefore, the value of m should be obtainable from the gradient of the linear plot of $\log N_a$ against $\log \Delta P$ implied by this equation.

Alternatively, values of m and n can be obtained by the direct measurement of growth rates in the various tests at a fixed crack length, a. The latter condition will ensure that the growth rates will be compared at a fixed value of $\phi(a/w)$ and hence for these growth rates the equation

$$\left.\frac{da}{dN}\right|_a = \text{const.}\Delta P^m P_{max}^n \quad (8)$$

will be appropriate. Thus, for the series of tests performed at a fixed value of ΔP but different P_{max} values, a plot of $\log (da/dN)|_a$ against $\log P_{max}^n$ will yield the value of n, and m can be obtained from a plot of $\log (da/dN)|_a$ against $\log \Delta P$, the data in the latter case being obtained from the series of tests carried out at constant P_{max} and different values of ΔP. In addition, since $\Delta P/P_{max} = 1 - R$, Equation 8 can be re-written as

$$\frac{da}{dN} = \frac{\text{const.}}{(1-R)^n} \Delta P^{(m+n)}. \quad (9)$$

Growth-rate data from tests performed at arbitrary R values should yield a straight line relationship if $\log (1-R)^n (da/dN)|_a$ is plotted against $\log \Delta P$ for a given crack length. Similarly the number of cycles required for growth to a given length, N_a, can be obtained from the plot of $\log N_a/(1-R)^n$ against $\log \Delta P$ (from the integration of Equation 9) and both these plots should display the same gradient (numerically), namely $(m+n)$. Of course, these plots can only be carried out if the value of n is already known but they provide useful corobarative evidence of the validity of the values of m and n obtained from the plots detailed earlier. In actual fact, if R is small (small P_{min}) and n is also small (~ 1) the variation in $(1-R)^n$ is not significant and its inclusion in Equation 9 only marginally affects the gradient of the plot, $(m+n)$.

Finally, the incorporation of the R-ratio in Equation 4 yields the expression:

$$\frac{da}{dN} = \frac{c'}{(1-R)^n}\Delta K^{(m+n)}, \qquad (10)$$

and so, the conventional Paris law analysis of a single a against N curve for a fixed R-ratio will, in fact, yield the term $(m+n)$ as the stress intensity range exponent, i.e. will include the K_{max} dependence.

4. Experimental results

The crack-growth data were obtained from three series of tests each characterized by different loading conditions. In the first series of tests the load range, ΔP, was fixed at 7 kg (nominally) but the tests were carried out at different values of P_{max} (and P_{min}). In the second series, P_{max} was fixed at 10 kg, and the tests were carried out at different values of ΔP by altering P_{min}. In the third series, P_{min} was fixed at 1 kg, and P_{max} was altered from test to test producing proportionate changes in ΔP as well. Each test was analysed individually according to the simple Paris equation (Equation 1) as described in Section 3.1 and Fig. 3 shows two of the better examples of log (da/dN) against log ΔK plots obtained by analysis of the a against N curves indicated. Values of m and c obtained from plots of this type, however, were rather variable, m being found to lie in the range 5 to 7 depending on the test under consideration. As it was difficult to detect any systematic correlation between the values of c and m and the loading conditions, an attempt was made to rationalize the results by obtaining the values for the rate of growth parameters from the selective combination of crack-growth rate data from groups of tests, rather than from each test considered separately, using the procedures described in Section 3.2. The treatment of the results involved in accomplishing this is set out below.

Figs 4 to 6 show the crack-growth data derived from the first series of tests for which ΔP is effectively constant and P_{max} is varied from test to test, each of the figures relating to a specific value of P_{max}. The curves show considerable scatter between specimens tested under nominally identical conditions but, in spite of this, the results clearly demonstrate that increasing P_{max} leads to an increase in the rate of growth of the cracks when ΔP is held constant.

Analysis of this variation in terms of Equation 6 is illustrated in Fig. 7 for which a fixed crack length of 25 mm was selected and the number of cycles required for crack growth to this length, N_{25} (from the common starting length of 20 mm) is plotted as a function of P_{max}. The values of N_{25} were obtained by averaging the results obtained at each P_{max} value and an additional data point, $P_{max} = 9$ kg, has been included in the figure. Although the paucity of the data does not allow the gradient of the plot to be established with confidence, a regression line through the points yielded a gradient close to 1 and the line fitted to the data in Fig. 7 has been drawn with a slope equal to 1, i.e., the value of n in Equation 4 is indicated to be 1.

The results of the second series of tests involving

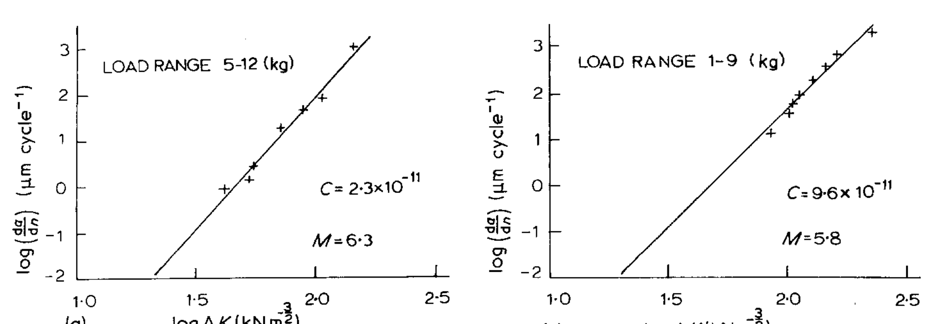

Figure 3 Paris law plots of the crack-growth data obtained from specimens cycled between 5 and 12 kg, and 1 and 9 kg, respectively.

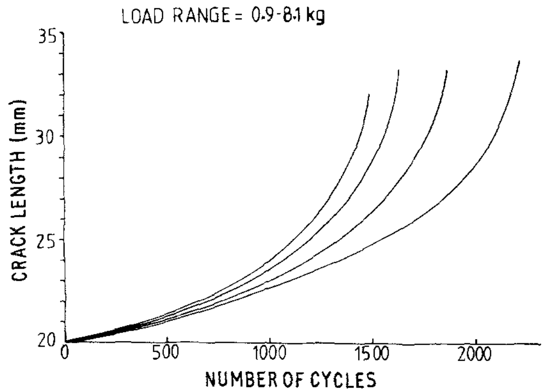

Figure 4 a against N curves for specimens tested at $\Delta P =$ 7 kg and $P_{max} = 8$ kg (nominal).

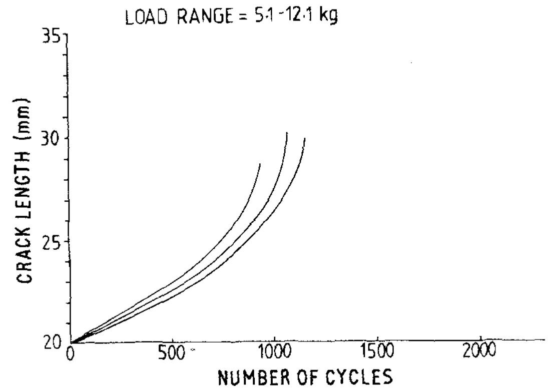

Figure 6 a against N curves for specimens tested at $\Delta P =$ 7 kg and $P_{max} = 12$ kg (nominal).

constancy of P_{max} for each test but different values of ΔP are shown in Fig. 8. Again a fixed crack length of 25 mm was selected for the purposes of analysing the data and crack-growth rates at this crack length, $(\mathrm{d}a/\mathrm{d}N)|_{25}$, and the number of cycles required for crack growth to this length, N_{25}, were measured for each ΔP value and plotted on a log–log basis as a function of ΔP. The resulting plots are shown in Fig. 9, an additional data point for $\Delta P = 5$ kg being included. A regression line through the growth-rate data has a gradient close to (just less than) 4 and the lines drawn through both sets of data have been drawn with gradients exactly equal to 4. According to Equation 7 (for the number of cycles required for growth to 25 mm) and Equation 8 (for the variation of growth rate at 25 mm) under these test conditions the slopes of both the plots should be (numerically) equal and should yield the value of m, indicated by the plots to equal 4. It is worth noting that the line drawn through the data points for the $\log N_{25}$ against $\log \Delta P$ plot could be made to fit the data and yield the indicated gradient of 4 only by ignoring the point corresponding to the lowest ΔP value (5 kg) and it was positioned on this basis. The justification for doing this will be discussed in the next section.

The third series of tests represented by the a against N curves shown in Fig. 10, and which involved changes in both ΔP and P_{max} from test to test, can be analysed in terms of Equation 9 and the two associated plots $\log (1-R)^n (\mathrm{d}a/\mathrm{d}N)|_{25}$ against $\log \Delta P$ and $\log N_{25}/(1-R)^n$ against $\log \Delta P$ are shown in Fig. 11, n having been taken (from Fig. 7) as being 1. Again, a regression line was fitted to the growth-rate data and the slope was found to be close to 5. The lines shown though both sets of data points, therefore, were drawn with gradients exactly equal to 5. As with the data shown in Fig. 9 it was necessary to ignore the data point corresponding to $\Delta P = 5$ kg in locating the $\log N_{25}/(1-R)^n$ against $\log \Delta P$ line to fit the data and retain the gradient of 5 and again the impli-

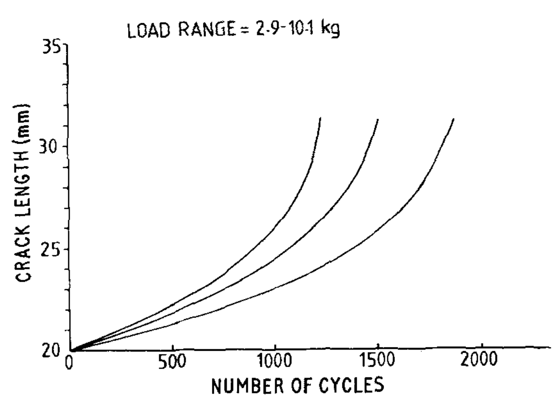

Figure 5 a against N curves for specimens tested at $\Delta P =$ 7 kg and $P_{max} = 10$ kg (nominal).

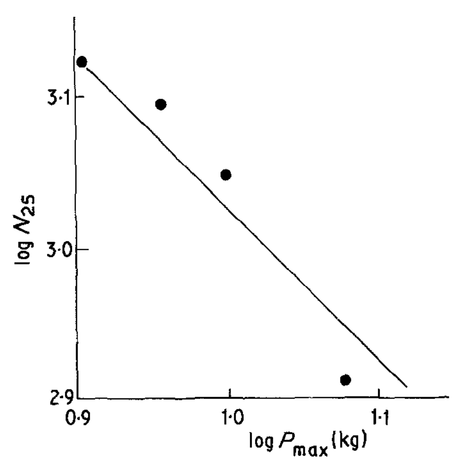

Figure 7 Log (number of cycles for growth to 25 mm), N_{25}, against $\log P_{max}$ for the data in Figs 4 to 6.

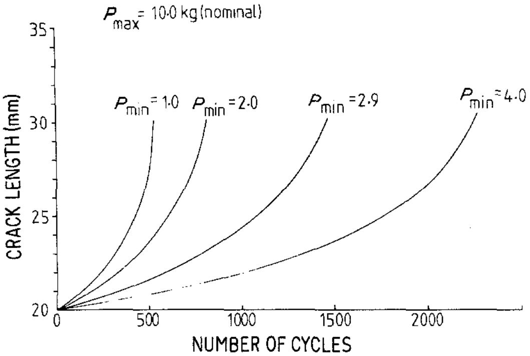

Figure 8 a against N curves obtained at $P_{max} = 10$ kg and the indicated ΔP values.

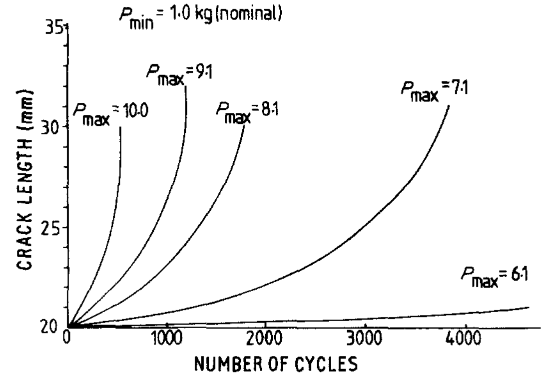

Figure 10 a against N curves for specimens tested between $P_{min} = 1$ kg and the P_{max} values indicated.

cations of the necessity for doing this will be discussed subsequently. Since the gradient of 5 indicated by Fig. 11 represents the "composite" exponent $(m + n)$ in Equations 9 and 10, the results obtained from the three groupings of the a against N data appear to be self consistent.

5. Discussion

The results presented in the previous section suggest that the application of the fracture mechanics approach to fatigue crack growth in polyurethane foam not only provides a basis for rationalizing the crack-growth data but also enables the effects of R-ratio to be dealt with on a reasonably quantitative basis. Of course, in a material which is inherently as variable in structure as a closed cell foam, it is unreasonable to expect a high degree of reproducibility in mechanical properties and the scatter in the a against N curves obtained under nominally identical conditions testifies to this aspect of the material. In the circumstances, therefore, the conformity of the crack-growth behaviour to either the simple Paris equation or the modified version including the effects of R-ratio is both gratifying and encouraging from the point of view of establishing design parameters. In this respect it is felt that the assignment of integral values to the crack-growth parameters m and n is justifiable even though the accuracy of the data presented here does not permit these values to be established with such certainty.

Although the analysis of the effect of maximum load on rate of growth, as represented by the $\log N_{25}$ against $\log P_{max}$ plot of Fig. 7, is based on only four points, the value of m obtained from Fig. 9 (i.e. 4), and the value of $m + n$ (= 5), obtained from Fig. 11 provide independent confirmation that the inference that $n = 1$, drawn from the data in Fig. 7, is justified. Of course, ideally the conventional Paris law analysis of the individual a against N curves for all the specimens tested should also yield values of stress intensity

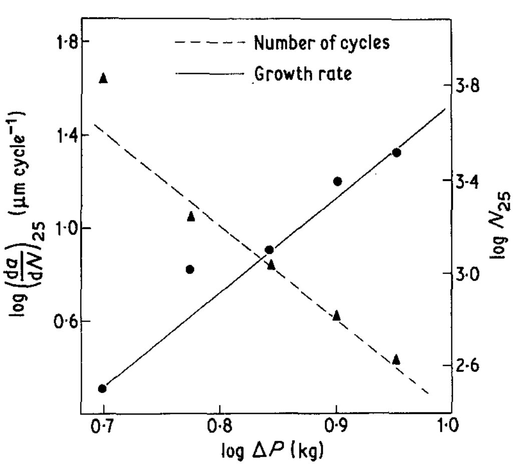

Figure 9 Log growth rate (at $a = 25$ mm) against log ΔP and $\log N_{25}$ against log ΔP for the data in Fig. 8.

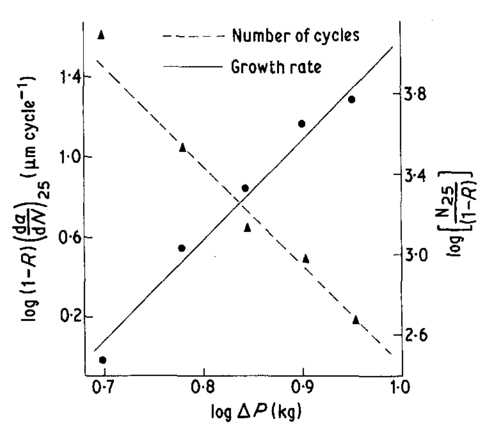

Figure 11 Log $(1 - R)(da/dN)|_{25}$ plotting against log ΔP and $\log N_{25}/(1 - R)$ plotted against log ΔP for the data shown in Fig. 10.

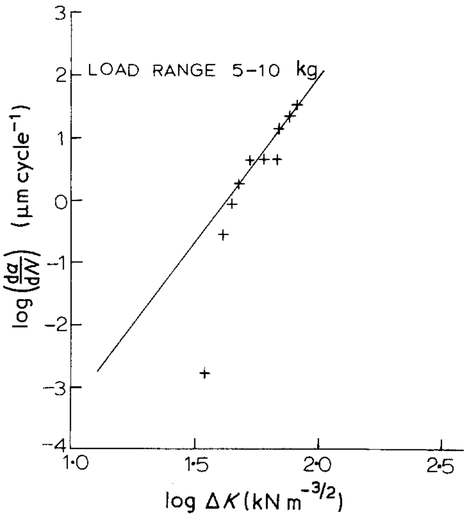

Figure 12 Log (da/dN) against log ΔK plot for $\Delta P = 5$ kg, $P_{max} = 10$ kg.

exponent equal to 5, according to Equation 10, and as noted in the previous section this was not found to be the case, the values obtained lying, usually, between 5 and 7. However, it is felt that this range of values is indicative of the inaccuracies involved in monitoring the progress of a single crack and of the variability of the material through which it propagates. In fact all the a against N curves could be fitted with fair accuracy to the numerically integrated form of Equation 10 with $(m + n) = 5$ and $c' = 2.85 \times 10^{-9}$, a being measured in μm and ΔK in kN m$^{-3/2}$. It is also worth pointing out that the method of analysis employed in establishing m and n from the combined data of several tests does not depend on the form of $\phi(a/w)$, only on its constancy for constant a while, of course, the analysis of the data from a single test does require such a knowledge of $\phi(a/w)$.

Although the plots from which the values of m and n were obtained all refer to crack-growth data measured at a fixed crack length of $a = 25$ mm, the form of the plots and the conclusions drawn are, within limits, not affected by which fixed crack length is used as the basis for comparison. However, the deviations of the points corresponding to $\Delta P = 5$ kg from the linear plots of $\ln N_{25}$, and $\ln N_{25}/(1 - R)$ against ΔP in Figs 9 and 11, respectively, are taken to be indicative of a significant change in behaviour at low ΔK values. The observed deviations of these points from the behaviour expected from the linear extrapolation of the higher ΔP data is in the direction of longer-than-expected crack "life" in both cases, the number of cycles required for growth to 25 mm being anomalously high. The anomalously low growth rates which this implies, however, were not apparent in the growth-rate measurements for these specimens at $a = 25$ mm suggesting that any such effect must only be manifesting itself in the very early stages of development of the crack, at the lowest ΔK values which, for these specimens, are the lowest employed in this series of tests. If this is so the anomalously low growth rates should be detectable in the plots of log (da/dN) against log ΔK derived from the a against N curves for the individual specimens in question. This was indeed the case and Fig. 12 shows such a plot for the specimen cycled between 5 and 10 kg in which the departure from the simple Paris growth law in the direction of 'abnormally' slow growth rates at the lowest ΔK values, in the first few millimeters of crack growth, is apparent. It appears, therefore, that there is a departure from the Paris growth law (as represented either by Equation 1 or 10) at low ΔK values, of the order of 40 kN m$^{-3/2}$ for this material. Such behaviour would be qualitatively consistent with the similar departures from the Paris law exhibited by metals [11] which are also in the direction of anomalously slow growth rates and occur as the "threshold" ΔK value, associated with zero rate of growth, is approached. From the design point of view the departure of the growth-rate data from prediction is in the "right" direction since the application of Equation 10 to crack-growth rates at lower ΔK values will lead to over-estimates of fatigue crack growth and so to under-estimates of the likely fatigue life of a component providing, therefore, a conservative design criterion.

The range of crack growth rates studied in the present tests is roughly from about 10^{-3} mm cycle^{-1} to 10^{-1} mm cycle. Since the average cell diameter of the foam is about 180 μm this implies that all the growth curves obtained relate to growth rates of much less than 1 cell diameter advance per cycle and indeed this figure is only approached in the very latest stages of crack growth as the growth becomes unstable. The stress intensity at which unstable growth occurs has not been accurately established by the present experiments but appears to correspond to the achievement of a stress intensity maximum of about 130 kN m$^{-3/2}$, although with considerable variability from specimen to specimen.

6. Conclusions

(1) Fatigue crack growth in polyurethane foam can be analysed in terms of fracture mechanics parameters and, for the foam with which the present work is concerned, the growth rate can be adequately expressed in terms of a modified form of the Paris equation written:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{c}{(1-R)}\Delta K^5. \qquad (11)$$

(2) The exponent of the stress intensity rate in the above equation reflects a fourth power dependence of growth rate on ΔK and a linear dependence on K_{max}.

(3) There is some evidence that the validity of the equation given in [11] breaks down at ΔK values of less than 40 kN m$^{-3/2}$, the measured values of growth rate being less than the predicted values.

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